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Long-range interaction between spins

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Abstract. It is shown that invariance of Lagrangian field theory under a class of the coordinate-dependent Lorentz group of transformations requires the introduction of a massless axial vector gauge field which gives rise to a super-weak long-range spin–spin force between particles in vacuum. Recent experiments demonstrating repulsion and attraction between circularly polarised laser beams are interpreted to be due to such a force enhanced by spin polarisation of sodium vapour, through which these beams pass.

1. Introduction

Gauge symmetry has been one of the most powerful tools for description of the fundamental processes in nature. In fact, in terms of present day convictions, it is the symmetry that dictates interaction between particles. To be more explicit, global charge gauge invariance of quantum field theory leads to conservation of charge in the sense that charge, counted in units of electronic charge, remains unaltered in particle reactions. On the other hand, local gauge invariance requires the introduction of a massless vector field, which leads to a long-range interaction between charges, thereby providing a mechanism for the measurement of charge. The physical consequence of invariance under the global SU(2) group of transformations is the conservation of isospin, which provides a framework for counting isospin and its projections; while invariance under local SU(2) transformation introduces the Yang–Mills fields which provide the necessary mechanism for dynamical measurement of isospin. Further extension of this theory leads to the explanation of the strong interaction in terms of the exchange of gluons.

The question we address ourselves to is whether spin can also be dynamically measured through the exchange of some gauge boson. It is to be recollected that global Lorentz invariance leads to conservation of angular momentum which provides a frame-work for the counting of spins and their projections. One would therefore expect that the local Lorentz invariance would provide the desired symmetry for the dynamical measurement of spin through the exchange of massless gauge bosons.

It is well known that invariance of the Lagrangian for Dirac particles under the coordinate-dependent Lorentz group of transformations requires a 24-component massless gauge field (Utiyama 1956, Kibble 1961, Salam 1973). These gauge fields, in combination with the 16-component vierbeins in Riemannian space, reproduce the standard theory of gravitation. We find that for a certain restricted coordinate dependence of the parameters of the Lorentz group of transformations, one needs a massless axial vector gauge field which gives rise to a force between particles with spin in flat space–time. This force is found to be repulsive for antiparallel spins and attractive for parallel spins. This is precisely the experimental result with the polarised laser beams. It has been observed by Tam and Happer (1977) that, while circularly polarised laser beams of opposite polarisation repel each other, beams of equal polarisations attract in a medium of sodium vapour for laser frequencies on the high-frequency wing of the D_1 line. In this paper, we interpret these results in terms of a long-range spin–spin force resulting from the exchange of massless axial vector gauge particles. The role of the sodium vapour, which is very crucial for the observation of the attraction and repulsion of the beams, is also explained. The spin–spin force is enhanced due to spin polarisation phenomena occurring in the sodium vapour. A generalised dielectric formulation of the enhancement of the interaction has been developed and the result for the deflection of the laser beams as a function of the sodium vapour density has been compared with experimental data. The agreement is excellent for coupling strength, $g^2/4\pi \simeq 10^{-9}$.

In § 2 we develop the gauge theory for spin–spin interaction. Section 3 deals with the calculation of the two-body spin–spin potential. The application of the theory in laser experiments is discussed in § 4.

2. Spin gauge theory

Under the Lorentz group of transformations

$$x_\mu \to x'_\mu = x_\mu + \alpha_{\mu\nu} x_\nu, \qquad \alpha_{\mu\nu} = -\alpha_{\nu\mu} \tag{1}$$

the Dirac field transforms as

$$\psi(x) \to \psi'(x') = \Omega\psi(x) \tag{2}$$

where

$$\Omega = \exp(\tfrac{1}{2}\mathrm{i}\Sigma_{\mu\nu}\alpha_{\mu\nu})$$

with $\Sigma_{\mu\nu} = \frac{1}{2}\mathrm{i}^{-1}\gamma_\mu\gamma_\nu$. The Dirac Lagrangian density

$$\mathcal{L}_\mathrm{D} = -\bar{\psi}(\gamma_\mu\partial_\mu + m)\psi \tag{3}$$

is invariant under this transformation, if $\alpha_{\mu\nu}$ are constants. However, if the parameters $\alpha_{\mu\nu}$ are coordinate dependent, this is not true any more. In this situation, one has to alter the Lagrangian to restore invariance. One defines the covariant derivative,

$$D_\mu\psi(x) = (\partial_\mu + \mathrm{i}gB_\mu(x))\psi(x) \tag{4}$$

where $B_\mu = B_{\mu,\nu\lambda}\Sigma_{\nu\lambda}$, $B_{\mu,\nu\lambda}$ being the well known 24-component field. This covariant derivative leads to the invariant Lagrangian,

$$\mathcal{L}_\mathrm{D} = -\bar{\psi}(\gamma_\mu D_\mu + m)\psi. \tag{5}$$

For the particular coordinate dependence

$$\varepsilon_{\mu\nu\lambda\zeta}\partial_\mu\alpha_{\nu\lambda} = 6\partial_\zeta\Lambda(x) \tag{6}$$

the Lagrangian (5) remains invariant, with

$$\begin{aligned}\gamma_\mu D_\mu\psi_r &= \gamma_\mu(\delta_{rs}\partial_\mu + \tfrac{1}{4}\mathrm{i}g\varepsilon_{\mu\nu\lambda\zeta}\Sigma_{\nu\lambda,rs}a_\zeta)\psi_s \\ &= (\gamma_\mu\partial_\mu + \tfrac{3}{4}g\gamma_\mu\gamma_5)\psi_r.\end{aligned} \tag{7}$$

The choice (6) implies complete antisymmetry in the indices of $\partial_\mu \alpha_{\nu\lambda}$ and corresponds to the reduction of the 24-component $B_{\mu,\nu\lambda}$ field to the four-component axial vector field a_μ, the fields being related by

$$B_{\mu,\nu\lambda}(x) = \tfrac{1}{4}\varepsilon_{\mu\nu\lambda\zeta} a_\zeta(x). \tag{8}$$

The law of transformation for the axial vector gauge fields follows from the general transformation law for the fields $B_\mu \equiv B_{\mu,\nu\lambda}\Sigma_{\nu\lambda}$ consistent with the invariance. B_μ transform as:

$$B_\mu(x) \to B'_\mu(x') = \Omega(x) B_\mu(x) \Omega(x)^{-1} - (\mathrm{i}g)^{-1}(\partial_\mu \Omega(x))\Omega(x)^{-1} \tag{9}$$

which leads to

$$a_\mu(x) \to a'_\mu(x') = a_\mu(x) + \tfrac{2}{3}\alpha_{\mu\nu}(x) a_\nu(x) - g^{-1}\partial_\mu \Lambda(x). \tag{10}$$

In the above derivation use has been made of the commutation of the generators given by:

$$[\Sigma_{\mu\nu}, \Sigma_{\rho\sigma}] = \mathrm{i}(\delta_{\mu\rho}\Sigma_{\nu\sigma} - \delta_{\mu\sigma}\Sigma_{\nu\rho} + \delta_{\nu\sigma}\Sigma_{\mu\rho} - \delta_{\nu\rho}\Sigma_{\mu\sigma}). \tag{11}$$

On account of (7) $\mathscr{L}_{\mathrm{D}}$ of (5) becomes

$$\mathscr{L}_{\mathrm{D}} = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi - \tfrac{3}{4} g \bar{\psi}\gamma_\mu \gamma_5 \psi a_\mu. \tag{5a}$$

The covariant curl of this axial photon gauge field can be written as

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \mathrm{i}g[B_\mu, B_\nu]. \tag{12}$$

The gauge field Lagrangian, therefore, works out to be

$$\begin{aligned} \mathscr{L}_{\mathrm{G}} &= -\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} \\ &= -\tfrac{1}{2}(\partial_\mu a_\nu)(\partial_\mu a_\nu) - \tfrac{3}{16} g^2 (a_\mu a_\mu)^2. \end{aligned} \tag{13}$$

In deriving (13) we have used

$$\Sigma_{\mu\nu,rs} = -\mathrm{i}(\delta_{\mu r}\delta_{\nu s} - \delta_{\mu s}\delta_{\nu r}) \tag{14}$$

as generator components of the vector field and also have used the Lorentz condition

$$\partial_\mu a_\mu = 0. \tag{15}$$

Having obtained the coupling of the axial vector gauge field to the spin-$\frac{1}{2}$ Dirac field, we now consider its coupling to the neutral spin-1 field, where the transformation laws under the Lorentz group of transformations are:

$$\begin{aligned} A_r(x) &\to A'_r(x') = \Omega_{rs}(x) A_s(x) \\ \Omega_{rs}(x) &= \exp(\tfrac{1}{2}\mathrm{i}\Sigma_{\mu\nu,rs}\alpha_{\mu\nu}(x)). \end{aligned} \tag{16}$$

The invariant Lagrangian for this massive vector field is given by

$$\mathscr{L}_{\mathrm{M}} = -\tfrac{1}{4}(D_\mu A_\nu - D_\nu A_\mu)(D_\mu A_\nu - D_\nu A_\mu) + m^2 A_\mu A_\mu \tag{17}$$

where

$$D_\mu A_r = \partial_\mu A_r + \tfrac{1}{4}\mathrm{i}g\varepsilon_{\mu\nu\lambda\zeta}\Sigma_{\nu\lambda,rs} a_\zeta A_s. \tag{18}$$

Upon simplification (17) reduces to

$$\begin{aligned} \mathscr{L}_{\mathrm{M}} = -\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - g\varepsilon_{\mu\nu\lambda\zeta}(\partial_\mu A_\nu) A_\lambda a_\zeta \\ - \tfrac{1}{2} g^2 [a_\mu a_\mu A_\nu A_\nu - A_\mu a_\mu A_\nu a_\nu] + m^2 A_\mu A_\mu. \end{aligned} \tag{19}$$

For the coupling of the photon field to the axial vector gauge field we have to set $m = 0$ in the above Lagrangian. But, equation (19), even without the mass term, is not invariant under the U(1) gauge transformation in $A_\mu \to A_\mu + \mathrm{e}^{-1}\alpha(x)$, where $\Box\alpha(x) = 0$. This can be rectified by expanding the theory so as to include a scalar field. It will be noticed that the lack of U(1) gauge invariance can be traced to the definition of covariant derivative $D_\mu A_r$ in equation (18). We therefore define a new covariant derivative

$$D_\mu A_r = \partial_\mu(A_r + \lambda\,\partial_r\phi) + \tfrac{1}{4}\mathrm{i}g\varepsilon_{\mu\nu\lambda\zeta}\Sigma_{\nu\lambda,rs}a_\zeta(A_s + \lambda\,\partial_s\phi) \tag{20}$$

where ϕ is the scalar field which transforms as

$$\phi(x) \to \phi'(x) = \phi(x) - (\lambda\,\mathrm{e})^{-1}\alpha(x)$$

under U(1). It may be seen that this new covariant derivative is invariant under U(1) and covariant under local Lorentz transformation. Using this covariant derivative one obtains the following invariant Lagrangian:

$$\mathscr{L} = -\tfrac{1}{4}\{f_{\mu\nu}f_{\mu\nu} + 2g\varepsilon_{\mu\nu s\zeta}f_{\mu\nu}a_\zeta(A_s + \lambda\,\partial_s\phi) + 2g^2[(A_s + \lambda\,\partial_s\phi)^2a^2 - ((A_s + \lambda\,\partial_s\phi)a_s)^2]\} \tag{21}$$

where $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. To this we have to add the kinetic energy term, $-\tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi)$, of the scalar field along with a counter term $-\lambda^{-1}(\partial_\mu\phi)A_\mu$ to ensure U(1) invariance of the kinetic energy term. Thus the complete Lagrangian reads

$$\begin{aligned}\mathscr{L} = -\tfrac{1}{4}\{&f_{\mu\nu}f_{\mu\nu} + 2g\varepsilon_{\mu\nu s\zeta}f_{\mu\nu}a_\zeta(A_s + \lambda\,\partial_s\phi) + 2g^2[(A_s + \lambda\,\partial_s\phi)^2a^2 - ((A_s + \lambda\,\partial_s\phi)a_s)^2]\}\\ &-\tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) - \lambda^{-1}(\partial_\mu\phi)A_\mu - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}\end{aligned} \tag{22}$$

where the last term stands for the axial field Lagrangian. Equation (22) can be rewritten as

$$\mathscr{L} = -\tfrac{1}{4}f_{\mu\nu}f_{\mu\nu} + J_\mu(A_\mu + \lambda\,\partial_\mu\phi) - \tfrac{1}{2}(\partial_\mu\phi)(\partial_\mu\phi) - \lambda^{-1}(\partial_\mu\phi)A_\mu - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{23}$$

where

$$J_K = -\tfrac{1}{2}g\varepsilon_{\mu\nu K\zeta}f_{\mu\nu}a_\zeta - \tfrac{1}{2}g^2[a^2(A_K + \lambda\,\partial_K\phi) - ((A_s + \lambda\,\partial_s\phi)a_s)a_K].$$

The equation of motion for the scalar field is

$$\Box\phi = \lambda\,\partial_\mu J'_\mu \tag{24}$$

where

$$J'_K = J_K + J''_K$$

with

$$J''_K = -\tfrac{1}{2}g^2[a^2(A_K + \lambda\,\partial_K\phi) - ((A_s + \lambda\,\partial_s\phi)a_s)a_K].$$

Finally in the limit $\lambda \to 0$

$$\mathscr{L} = -\tfrac{1}{4}f_{\mu\nu}f_{\mu\nu} + A_\mu(\delta_{\mu\nu} - \partial_\mu\partial_\nu/\Box)J_\nu - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{25}$$

where we have thrown a four divergence term and used the Lorentz condition for the vector fields. In this limit,

$$J_K = -\tfrac{1}{2}g\varepsilon_{\mu\nu K\zeta}f_{\mu\nu}a_\zeta - \tfrac{1}{2}g^2(A_Ka^2 - (a\cdot A)a_K). \tag{26}$$

The additional term $(\partial_\mu\partial_\nu J_\nu/\Box)$ appearing in the above Lagrangian restores the U(1) gauge invariance without in any way affecting the relevant physics, since on account of

$\partial_\mu A_\mu = 0$, it contributes a four divergence term to the Lagrangian. This term is non-local; however, it should be remembered that it is obtained as the limiting case of a local theory.

It is to be noted that while dealing with symmetries of Lagrangian field theories, it is the action which is required to be invariant, $\delta S = S' - S = 0$. For the case of gauging the Lorentz group (Kibble 1961) one has

$$S = \int \mathrm{d}^4 x\, \mathscr{L}(x)$$

and

$$S' = \int \mathrm{d}^4 x' \mathscr{L}'(x') = \int \mathrm{d}^4 x\, \mathscr{L}'(x') J \tag{27}$$

where J is the Jacobian for transformation of the four volume element. So, invariance of the action implies $\mathscr{L}'(x')[1 + \partial_\mu(\delta x_\mu)] = \mathscr{L}(x)$ which is identical to $\mathscr{L}'(x) = \mathscr{L}(x)$. One has, therefore, to consider a group of transformations in the local tangent space where the spinor fields transform as $\psi'(x) = \Omega(x)\psi(x)$, where $\Omega(x)$ is the SL(2, C) double covering of $\Lambda(x)$ and the vierbeins $h_{\mu a}$ required in the definition of covariant derivatives transform as $h'_{\mu a}(x) = h_{\mu b}(x)\Lambda^b_a(x)$. But in our case of gauging the Lorentz group with the particular choice of the transformation parameters, given by equation (6), it turns out that $\partial_\mu(\delta x_\mu) = 0$; so that the Jacobian is unity. This suggests that the invariance of the action S for our case requires $\mathscr{L}'(x') = \mathscr{L}(x)$, which is different from the requirement $\mathscr{L}'(x) = \mathscr{L}(x)$ of the conventional 'local Lorentz group' in general relativity.

Before concluding this section a further point is in order. Since the local Lorentz transformations for arbitrary coordinate dependence of $\alpha_{\mu\nu}(x)$ form a group, it should be evident that any particular coordinate dependence of $\alpha_{\mu\nu}(x)$ should also form a group.

3. Two-body potential

The two-body potential due to a single axial photon exchange can be calculated using the interaction Lagrangians. In the following we calculate the potentials between (i) two fermions (ii) two vectors and (iii) a vector boson and a fermion. The potential will be defined as the Fourier transform of the non-relativistic limit of the scattering amplitude in the single axial photon-exchange approximation, when no energy, but only momentum, is transferred.

The Feynman diagram for the scattering of two fermions due to exchange of a single axial photon is given in figure 1. The derivation of the interaction potential is carried out in the standard way (Berestetskii *et al* 1971). The scattering amplitude is given by

$$M_{fi} = g'^2(\bar{u}'_1\gamma_\mu\gamma_5 u_1)D_{\mu\nu}(q)(\bar{u}'_2\gamma_\nu\gamma_5 u_2) \tag{28}$$

where

$$q = p'_1 - p_1 = p_2 - p'_2 \qquad \text{and} \qquad g' = \tfrac{3}{4}g.$$

The use of the expansion of the spinor function,

$$u(p) = \sqrt{2m}\begin{pmatrix} (1 - |\boldsymbol{p}|^2/8m^2c^2)W \\ (\boldsymbol{\sigma}\cdot\boldsymbol{p}/2mc)W \end{pmatrix},$$

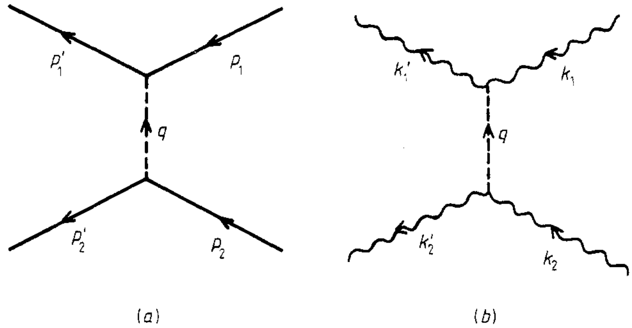

Figure 1. (*a*) Feynman diagram for scattering of two fermions in the single-axial photon-exchange approximation. The broken line represents the axial photon propagator. (*b*) Feynman diagram for scattering of two photons. The wavy lines stand for the photons.

and the Coulomb gauge for the axial vector propagator,

$$D_{00} = -4\pi/\boldsymbol{q}^2, \qquad D_{0i} = 0, \qquad D_{ik} = \frac{4\pi}{\boldsymbol{q}^2 - \omega^2/c^2}(\delta_{ik} - q_i q_k/\boldsymbol{q}^2),$$

in the non-relativistic limit $c^{-1} \to 0$ gives

$$M_{fi} = -2m_1 2m_2 (W_1'^* W_2'^* U(\boldsymbol{q}) W_1 W_2) \tag{29}$$

where W are the two-component objects satisfying $W^* W = 1$. Here

$$U(\boldsymbol{q}) = -\frac{4\pi g'^2}{\boldsymbol{q}^2}\left((\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) - \frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{\boldsymbol{q}^2}\right). \tag{30}$$

The Fourier transform of the potential (30) gives

$$V_{ab}^F(\boldsymbol{r}) = -\frac{g'^2}{2r}\left((\boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b) + \frac{(\boldsymbol{\sigma}_a \cdot \boldsymbol{r})(\boldsymbol{\sigma}_b \cdot \boldsymbol{r})}{r^2}\right). \tag{31}$$

Before proceeding to calculate the interaction potential between two vector particles, which also includes photons, we may recall that in QED, photon–photon scattering takes place in the fourth order of the coupling constant. In contrast, the exchange of the axial photon predicts the possibility of photon–photon scattering in second order of the coupling strength. The Feynman diagram for the process is given by figure 1(*b*). The scattering amplitude reads

$$M_{fi} = -g^2 \varepsilon_{\mu\nu\lambda\zeta}\varepsilon_{\mu'\nu'\lambda'\zeta'} e_{1\nu}' e_{2\nu'}' (2k_1 + q)_\mu (2k_2 - q)_{\mu'} e_{1\lambda} e_{2\lambda'} D_{\zeta\zeta'}(q). \tag{32}$$

Using the non-relativistic propagator for the axial photon in the propagator gauge and the definition of the spin matrix $(S_i)_{ab} = -\mathrm{i}\varepsilon_{abi}$, which leads to the identity

$$\langle \boldsymbol{e}' | S_i | \boldsymbol{e} \rangle = -\mathrm{i}(\boldsymbol{e}' \times \boldsymbol{e})_i, \tag{33}$$

the amplitude turns out to be

$$M_{fi} = 4g^2 \omega_1 \omega_2 \frac{4\pi}{\boldsymbol{q}^2} \langle \boldsymbol{e}_1' \boldsymbol{e}_2' | [(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2) - (\boldsymbol{S}_1 \cdot \boldsymbol{q})(\boldsymbol{S}_2 \cdot \boldsymbol{q})/\boldsymbol{q}^2] | \boldsymbol{e}_1 \boldsymbol{e}_2 \rangle. \tag{34}$$

From (34), the two-body potential follows as:

$$V_{ab}^{M}(\boldsymbol{r}) = -\frac{g^2}{2r}\left((\boldsymbol{S}_a \cdot \boldsymbol{S}_b) + \frac{(\boldsymbol{S}_a \cdot \boldsymbol{r})(\boldsymbol{S}_b \cdot \boldsymbol{r})}{r^2}\right). \tag{35}$$

A similar procedure for scattering between a fermion and a vector boson (neutrino and photon) leads to the interaction potential,

$$V^{FM}(\boldsymbol{r}) = -\frac{3g^2}{8r}\left((\boldsymbol{\sigma} \cdot \boldsymbol{S}) + \frac{(\boldsymbol{\sigma} \cdot \boldsymbol{r})(\boldsymbol{S} \cdot \boldsymbol{r})}{r^2}\right). \tag{36}$$

It is clear from the equations (31), (35) and (36) that the force between two particles with spins is of long range and Coulomb-like. It is attractive for parallel spins and repulsive for antiparallel ones.

4. Experiment with polarised laser beams

The above long-range spin–spin interaction must be very weak, otherwise, it would contribute substantially to electrodynamic processes, such as Moller scattering, and Compton scattering etc. Since our claim is that this very weak interaction is responsible for the observed attraction and repulsion between polarised laser beams in sodium vapour, it is necessary to explain the mechanism of its enhancement due to spin-polarised vapour atoms. The circularly polarised laser beam with a frequency slightly higher than that of the sodium D_1 line optically pumps the atoms from the ground state to the $3^2P_{1/2}$ state, thereby producing a medium of spin-polarised atoms. The spins of the two circularly polarised laser beams interact with each other through this medium. We outline a linear response theory for the 'dielectric function' $\tilde{\varepsilon}(\boldsymbol{q}, 0)$, that determines the effective interaction between photons in such a medium, in terms of the corresponding interaction in vacuum. The effective scattering amplitude is given by

$$F(\boldsymbol{q}) = f(\boldsymbol{q})/\tilde{\varepsilon}(\boldsymbol{q}, 0) \tag{37}$$

$f(\boldsymbol{q})$ being the amplitude for scattering of two polarised photons in vacuum, in the one axial photon-exchange approximation. The virtual axial photon emitted by the laser photon interacts with the spin density of the spin polarised medium producing an induced spin density, which in the linear approximation is given by

$$\langle S_i^{\text{ind}}(\boldsymbol{x}, t)\rangle = -\mathrm{i}n\int \mathrm{d}^3x' \int \mathrm{d}t'\,\theta(t-t')\langle 0|[S_i(\boldsymbol{x}, t), S_j(\boldsymbol{x}', t')]|0\rangle a_j(\boldsymbol{x}', t'). \tag{38}$$

Here

$$S_i(\boldsymbol{x}, t) = (\boldsymbol{\nabla}^2 - \partial^2/\partial t^2)a_i(\boldsymbol{x}, t) \tag{39}$$

and n is the density of the sodium atoms. The state $|0\rangle$ stands for the state $3^2P_{1/2}$ of the sodium atom. Equation (38) can be converted to

$$\langle S_i^{\text{ind}}(\boldsymbol{x}, t)\rangle = -\mathrm{i}n\int \mathrm{d}^3x' \int \mathrm{d}t'\,\theta(t-t')\langle 0|[S_i(\boldsymbol{x}, t), a_j(\boldsymbol{x}', t')]|0\rangle S_j(\boldsymbol{x}', t') \tag{40}$$

with the aid of equation (39) followed by integration by parts. Inside the medium

$$S_i^{\text{total}}(\boldsymbol{x}, t) = S_i^{\text{ext}}(\boldsymbol{x}, t) + \langle S_i^{\text{ind}}(\boldsymbol{x}, t)\rangle$$
$$= \int d^3x' \int dt' \, \varepsilon_{ij}(\boldsymbol{x}-\boldsymbol{x}', t-t') S_j^{\text{ext}}(\boldsymbol{x}', t'). \tag{41}$$

To evaluate $\tilde{\varepsilon}_{ij}(\boldsymbol{q}, \omega)$ we introduce a complete set of intermediate states in equation (40), so that

$$\tilde{\varepsilon}_{ij}(\boldsymbol{q}, \omega) = \delta_{ij} - in \sum_r \int_0^\infty dt \, e^{i\omega t}[\langle 0|S_i(\boldsymbol{q}, 0)|r\rangle\langle r|a_j(-\boldsymbol{q}, -t)|0\rangle$$
$$-\langle 0|a_j(-\boldsymbol{q}, -t)|r\rangle\langle r|S_i(\boldsymbol{q}, 0)|0\rangle]$$

which on using equation (39) becomes

$$\tilde{\varepsilon}_{ij}(\boldsymbol{q}, \omega) = \delta_{ij} + in \int_0^\infty dt \, e^{i\omega t} \sum_r [\langle 0|S_i(\boldsymbol{q}, 0)|r\rangle\langle r|S_j(-\boldsymbol{q}, -t)|0\rangle$$
$$-\langle 0|S_j(-\boldsymbol{q}, -t)|r\rangle\langle r|S_i(\boldsymbol{q}, 0)|0\rangle]\frac{1}{\boldsymbol{q}^2 - (\omega_0 - \omega_r)^2}. \tag{42}$$

On carrying out the time integration, we obtain

$$\tilde{\varepsilon}_{ij}(\boldsymbol{q}, 0) = \delta_{ij} - n \sum_r [\langle 0|S_i(\boldsymbol{q}, 0)|r\rangle\langle r|S_j(-\boldsymbol{q}, 0)|0\rangle + \langle 0|S_j(-\boldsymbol{q}, 0)|r\rangle\langle r|S_i(\boldsymbol{q}, 0)|0\rangle]$$
$$\times \{(-\omega_0 + \omega_r + i\eta)[\boldsymbol{q}^2 - (\omega_0 - \omega_r)^2]\}^{-1}. \tag{43}$$

Because of the close proximity of the state $3^2P_{3/2}$ to the state $3^2P_{1/2}$, this intermediate state will give the most dominant contribution on account of the energy denominator. We therefore ignore all the other intermediate state contributions and obtain

$$\tilde{\varepsilon}_{ij}(\boldsymbol{q}, 0) = \delta_{ij} - \frac{nc_{ij}(\boldsymbol{q}, 0)}{\Omega(\boldsymbol{q}^2 - \Omega^2)} \tag{44}$$

where

$$C_{ij}(\boldsymbol{q}, 0) = [\langle 3^2P_{1/2}|S_i(\boldsymbol{q}, 0)|3^2P_{3/2}\rangle\langle 3^2P_{3/2}|S_j(-\boldsymbol{q}, 0)|3^2P_{1/2}\rangle$$
$$+ \langle 3^2P_{1/2}|S_j(-\boldsymbol{q}, 0)|3^2P_{3/2}\rangle\langle 3^2P_{3/2}|S_i(\boldsymbol{q}, 0)|3^2P_{1/2}\rangle]$$

and

$$\Omega = E(3^2P_{3/2}) - E(3^2P_{1/2}).$$

Since in the limit $\boldsymbol{q} \to 0$, $S_i(\boldsymbol{q}, 0) = gS_i(0)$

$$C_{ij}(0) = g^2[\langle 3^2P_{1/2}|S_i(0)|3^2P_{3/2}\rangle\langle 3^2P_{3/2}|S_j(0)|3^2P_{1/2}\rangle$$
$$+ \langle 3^2P_{1/2}|S_j(0)|3^2P_{3/2}\rangle\langle 3^2P_{3/2}|S_i(0)|3^2P_{1/2}\rangle].$$

Instead of the Hermitian basis, it is more convenient to use the canonical basis for spins:

$$C_{ij}(\boldsymbol{q}) = 2\delta_{ij}C_{-+}(\boldsymbol{q}) \tag{45}$$

where

$$C_{-+} = g^2[\langle\tfrac{1}{2}, -\tfrac{1}{2}|S_-|\tfrac{1}{2}, \tfrac{1}{2}\rangle\langle\tfrac{1}{2}, \tfrac{1}{2}|S_+|\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \langle\tfrac{1}{2}, -\tfrac{1}{2}|S_+|\tfrac{1}{2}, \tfrac{1}{2}\rangle\langle\tfrac{1}{2}, \tfrac{1}{2}|S_-|\tfrac{1}{2}, -\tfrac{1}{2}\rangle] = g^2.$$

So the relevant dielectric function is

$$\tilde{\varepsilon}(\boldsymbol{q}, 0) = 1 - 2ng^2/\Omega(\boldsymbol{q}^2 - \Omega^2). \tag{46}$$

The photon–photon scattering amplitude, in a medium of sodium atoms under conditions discussed above, therefore becomes, for small $\boldsymbol{q}$,

$$F(\boldsymbol{q}) = f(\boldsymbol{q})[1 - 2ng^2/\Omega(\boldsymbol{q}^2 - \Omega^2)]^{-1}. \tag{47}$$

Although the light beam will be scattered in all directions, there will be a particular angle at which the scattering amplitude will have a peak; and we shall take this to be the direction of the scattered beam. This will correspond to $\boldsymbol{q}$ at which $\tilde{\varepsilon}(\boldsymbol{q}, 0)$ has a minimum. One finds that the angle θ between the incident and the scattered beam is given by

$$\theta_S = \left(\frac{\boldsymbol{q}^2}{\boldsymbol{k}^2}\right)^{1/2} = \frac{\Omega}{|\boldsymbol{k}|}\left(1 + \frac{2ng^2}{\Omega^3}\right)^{1/2} \tag{48}$$

where $\boldsymbol{k}$ is the momentum of the laser photon. This is somewhat different from the angle measured by Tam and Happer (1977). They measure half the angle of divergence between the scattered beams, the incident beams having a initial convergence. This angle is

$$\theta = \theta_S - \theta_0 = \frac{\Omega}{|\boldsymbol{k}|}\left(1 + \frac{2ng^2}{\Omega^3}\right)^{1/2} - \theta_0 \tag{49}$$

where $2\theta_0$ is the angle of convergence of the two incident laser beams. Using the experimental values

$2\theta_0 = 2$ mrad,

$|\boldsymbol{k}| = 16970\ \mathrm{cm}^{-1}$ for the sodium D_1 line and

$\Omega = 17\ \mathrm{cm}^{-1}$ for sodium $D_1 - D_2$ separation,

we find good agreement between the graph of θ against n and our result (49) for $g^2/4\pi = 0.7 \times 10^{-9}$. The results of our theory and the experimental observations of Tam and Happer (1977) are compared in figure 2. It will be seen that except at low densities the agreement is excellent.

It may be recalled that while reporting their experiment, Tam and Happer have outlined a theoretical explanation of their observations. We have closely examined their arguments and find that their semiclassical, semi-quantitative theory is unsatisfactory as well as incomplete. In the following we outline some of the more serious defects of their arguments.

These authors seem to base their theory on the belief that 'the force is transmitted between the beams by sodium atoms in much the same way that virtual photons are believed to transmit the force between the charged particles' (Happer and Tam 1977). If this were the case, the resulting force would not be of long range since atoms are massive.

Their explanation of the repulsion of laser beams is based on the formula

$$u = -\alpha^1\{|\mathscr{E}_-|^2 P_\uparrow + |\mathscr{E}_+|^2 P_\downarrow\} \tag{50}$$

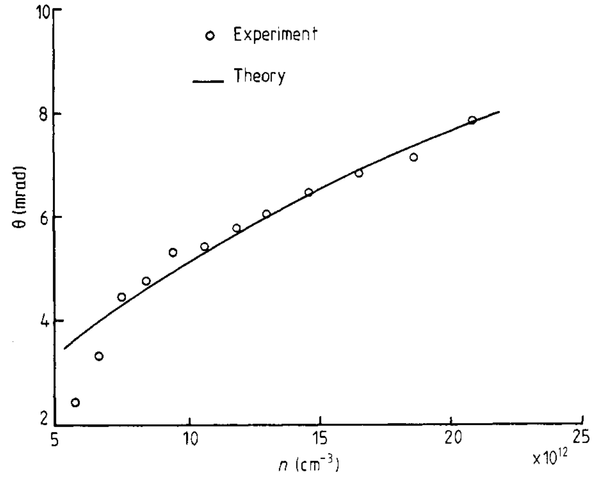

Figure 2. Comparison of the observed deviation angle θ with the predicted values for various sodium densities n.

which causes the expulsion of spin up (spin down) atoms from the $\sigma_+(\sigma_-)$ beams. As a result, the light beams bend under the force of reaction. Here one can raise the following two objections. Firstly, nothing debars the atoms from flying in all possible directions, In fact, figure 1 of Tam and Happer's paper (1977) depicts such a state of affairs. If this is so, the net reaction on the beams should vanish and the beams should not bend. Secondly, even if one accepts a preferential flight of the atoms, along the direction of the other beam, attraction between two σ_+ or two σ_- beams cannot take place. This is because, their interaction energy being positive, light beams can only expel atoms. This will always give rise to repulsion between light beams, irrespective of their polarisation.

The authors have ignored the initial convergence of the beams in their derivation of the angle between the emergent beams, the inclusion of which will appreciably affect the agreement of their final result with experiment.

It is worth noting that equation (47) in the limit $n \to 0$ leads to Coulomb-like scattering between two circularly polarised light beams. This is the crucial difference between our theory and that of Tam and Happer. However, due to the extremely feeble strength of the interaction, as estimated by us in the above, the deviation of the beams in vacuum may be undetectable.

5. Conclusion

We thus conclude that there exists an elementary interaction between spins of particles, which follows from a gauge principle. This force is found to be weaker than the weak interaction; it is of the same order of magnitude as the super-weak interaction (Wolfenstein 1964). We have examined all consequences of such an interaction on normal QED processes and seen that because of the abnormally weak coupling between spins, its contribution to quantum electrodynamic processes will be negligibly small. However, the weak spin–spin coupling, like gravitation, may manifest itself in astrophysical phenomena through a photon–neutrino interaction.

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